Polyakov Loop at Finite Temperature in Chiral Quark Models*

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Abstract

At finite temperature, chiral quark models do not incorporate large gauge invariance which implies genuinely non-perturbative finite temperature gluonic degrees of freedom. Motivated by this observation, we describe how the coupling of the Polyakov loop as an independent degree of freedom to quarks not only accounts for large gauge invariance, but also allows to establish in a dynamical way the interaction between composite hadronic states such as Goldstone bosons to finite temperature non-perturbative gluons in a medium which can undergo a confinement-deconfinement phase transition.

1 Large Gauge Transformations

One feature of gauge theories like QCD at finite temperatures in the imaginary time formulation [1, 2, 3] is the non-perturbative manifestation of the non Abelian gauge symmetry. In the Polyakov gauge, where $\partial_4 A_4 = 0$ and A_4 is a diagonal and traceless $N_c \times N_c$ matrix, and N_c is the number of colors, there is still some freedom in choosing the gluon field. Let us consider for instance the periodic gauge transformation [4, 5]

$$g(x_4) = e^{i2\pi x_4 \Lambda/\beta}\,, \tag{1}$$

where Λ is a color traceless diagonal matrix of integers. We call it a large gauge transformation (LGT) since it cannot be considered to be close to the identity[1]. The gauge transformation on the A_4 component of the gluon field is

$$A_4 \to A_4 + \frac{2\pi}{\beta}\Lambda\,. \tag{2}$$

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[1]Note that they are not large in the topological sense, as discussed in [4, 5].

Thus, invariance under the LGT, Eq. (1), implies a constant shift in the A_4 gluon amplitudes, meaning that A_4 is not uniquely defined by the Polyakov gauge condition. These ambiguities on the choice of the gauge field within a given gauge fixing are usually called Gribov copies. The requirement of gauge invariance actually implies identifying all amplitudes differing by a multiple of $2\pi/\beta$, which means periodicity in the diagonal amplitudes of A_4 of period $2\pi/\beta$. Perturbation theory, which corresponds to expanding in powers of small A_4 fields manifestly breaks gauge invariance at finite temperature, because a Taylor expansion on a periodic function violates the periodicity behavior. Thus, taking into account these Gribov replicas is equivalent to explicitly deal with genuine non-perturbative finite temperature gluonic degrees of freedom. A way of automatically taking into account LGT is by considering the Polyakov loop Ω as an independent variable, which in the Polyakov gauge becomes a diagonal unitary matrix

$$\Omega = e^{\mathrm{i}\beta A_4(\vec{x})} \tag{3}$$

invariant under the set of transformations given by Eq. (1). The relevance of the Polyakov loop in practical calculations is well recognized [1] but seldomly taken into account in high temperature calculations where large gauge invariance is manifestly broken since the gluon field is considered to be small. We have recently developed an expansion keeping these symmetries in general theories and applied it to QCD at the one quark+gluon loop level [6, 7].

2 The Center Symmetry

In pure gluodynamics, or in the quenched approximation (valid for heavy quarks) at finite temperature there is actually a larger symmetry since one can extend the periodic transformations to aperiodic ones [3],

$$g(x_4+\beta) = z g(x_4)\,, \qquad z^{N_c} = 1 \tag{4}$$

so that z is an element of the center $Z(N_c)$ of the group $SU(N_c)$. This center symmetry is a symmetry of the action as well as the gluon field boundary conditions. An example of such a transformation in the Polyakov gauge is given by

$$g(x_4) = e^{i2\pi x_4 \Lambda/N_c\beta}\,. \tag{5}$$

On the A_4 component of the gluon field produces

$$A_4 \to A_4 + \frac{2\pi}{N_c\beta}\Lambda\,. \tag{6}$$

Thus, in the quenched approximation the period is N_c times smaller than in full QCD. Under these transformations the gluonic action, measure and boundary conditions are invariant. The Polyakov loop, however, transforms as the fundamental representation of the $Z(N_c)$ group, i.e. $\Omega \to z\Omega$, yielding $\langle\Omega\rangle = z\langle\Omega\rangle$ and hence $\langle\Omega\rangle = 0$ in the unbroken center symmetry phase. At high temperatures one expects perturbation theory to hold, the gluon field amplitude becomes small and hence $\langle\Omega\rangle \to 1$, justifying the choice of Ω as an order parameter for a confinement-deconfinement phase transition. More generally, in the confining phase

$$\langle\Omega^n\rangle = 0 \qquad \text{for} \qquad n \neq mN_c \tag{7}$$

with m an arbitrary integer. The antiperiodic quark fields at the end of the Euclidean imaginary interval transform as $q(\vec{x},\beta) = -q(\vec{x},0) \to zq(\vec{x},\beta) = -q(\vec{x},0)$, so that the center symmetry is explicitly broken by the presence of dynamical quarks. A direct consequence of such a property is that, in the quenched approximation non-local condensates fulfill a selection rule of the form,

$$\langle \bar{q}(n\beta)q(0)\rangle = 0 \qquad \text{for} \qquad n \neq mN_c \tag{8}$$

since under the large aperiodic transformations given by Eq. (5) we have $\bar{q}(n\beta)q(0) \to z^{-n}\bar{q}(n\beta)q(0)$. This selection rule has some impact on chiral quark models.

3 Chiral quark models at finite temperature

To fully appreciate the role played by the center symmetry in chiral quark models (for a recent review on such models see e.g. Ref. [8] and references therein) let us evaluate the chiral condensate at finite temperature. At the one loop level one has[2]

$$\langle \bar{q}q\rangle^* \;=\; 4MT\mathrm{Tr}_c \sum_{\omega_n} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_n^2 + k^2 + M^2} \tag{9}$$

where $\omega_n = 2\pi T(n+1/2)$ are the fermionic Matsubara frequencies, M is the constituent quark mass and Tr_c stands for the color trace in the fundamental representation which in this case trivially yields a N_c factor. Possible finite cut-off corrections, appearing in the chiral quark models such as the NJL model at finite temperature have been neglected. This is a reasonable approximation as long as the temperature is low enough $T \ll \Lambda \sim 1\mathrm{GeV}$. The condensate can be rewritten as

$$\langle \bar{q}q\rangle^* \;=\; \sum_n (-1)^n \langle \bar{q}(n\beta)q(0)\rangle \tag{10}$$

in terms of nonlocal Euclidean condensates at zero temperature. After Poisson resummation, at low temperatures we have

$$\begin{aligned}
\langle \bar{q}q\rangle^* \;&=\; \langle \bar{q}q\rangle + 8N_c \sum_{n=1}^{\infty} (-)^n \frac{TM^2}{\pi^2} K_1(Mn/T)\,, \\
&\sim\; \langle \bar{q}q\rangle - \sum_{n=1}^{\infty} (-)^n \frac{N_c}{2} \left(\frac{2MnT}{\pi}\right)^{3/2} e^{-nM/T}\,,
\end{aligned} \tag{11}$$

where the asymptotic expansion of the modified Bessel function K_1 has been used. One can interpret the previous formula for the condensate in terms of statistical Boltzmann factors, since at large Euclidean coordinates the fermion propagator behaves as $S(i\beta,\vec{x}) \sim e^{-M\beta}$, so that we have contributions from multiquark states. This is a problem since it means that the heat bath is made out of free constituent quarks without any color clustering[3]. Another problem comes from comparison with Chiral Perturbation Theory

[2] We use an asterisk to denote finite temperature observables.

[3] One could think that this is a natural consequence of the lack of confinement in chiral quark models such as NJL. Contrary to naive expectations this is not necessarily the case; Boltzmann factors occur in quark models with analytic confinement such as the Spectral Quark Model [9]. There the condensate is

at Finite Temperature [10]. In the chiral limit, i.e., for $m_\pi \ll 2\pi T \ll 4\pi f_\pi$ the leading thermal corrections to the quark condensate are given by

$$\langle \bar{q}q \rangle^* \Big|_{\rm ChPT} = \langle \bar{q}q \rangle \left(1 - \frac{T^2}{8f_\pi^2} - \frac{T^4}{384 f_\pi^4} + \dots \right) . \tag{13}$$

This formula is derived under the assumption that there is no temperature dependence of the low energy constants, i.e. $L_i^* \simeq L_i$ so that the whole effect is due to thermal pion loops. Thus, the finite temperature correction is N_c-suppressed as compared to the zero temperature value. This *is not* what one sees in chiral quark model calculations; in the large N_c limit *there is* a finite temperature correction, which would mean that the low energy constants which appear in the chiral Lagrangian would have a genuine tree level temperature dependence, $L_i^* - L_i \simeq N_c e^{-M/T}$. To obtain the ChPT result of Eq. (13) pion loops have to be considered [11] and dominate for $T \ll M$. The problem is that already without pion loops chiral quark models predict a chiral phase transition at about $T_c \sim 170$ MeV, in remarkable but perhaps unjustified agreement with lattice calculations.

4 Coupling the Polyakov loop

In the Polyakov gauge one can formally keep track of large gauge invariance at finite temperature by coupling gluons to the model in a minimal way. This means in practice using the modified fermionic Matsubara frequencies [4, 5]

$$\hat{\omega}_n = 2\pi T(n + 1/2 + \nu) \, , \quad \nu = (2\pi i)^{-1} \log \Omega \tag{14}$$

which are shifted by the logarithm of the Polyakov loop which we assume for simplicity to be $\vec{x}$ independent. Previous work have coupled similarly Ω on pure phenomenological grounds [12, 13, 14], but the key role played by the implementation of large gauge invariance was not recognized. This is the only place where explicit dependence on colour degrees of freedom appear. This coupling introduces a colour source into the problem for a fixed A_0 field and projection onto the colour neutral states by integrating over the A_0 field, in a gauge invariant manner, as required. Actually, at the one quark loop level there is an accidental $Z(N_c)$ symmetry in the model which generates a similar selection rule as in pure gluodynamics, from which a strong thermal suppression, $\mathcal{O}(e^{-N_c M/T})$ follows. In this way compliance with ChPT can be achieved since now $L_i^* - L_i \simeq e^{-N_c M/T}$ but also puts some doubts on whether chiral quark models still predict a chiral phase transition at realistic temperatures. This question has been addressed using specific potentials for the Polyakov loop either based on one loop perturbation theory for massive gluons [13] in the high temperature approximation or on strong coupling expansions on the lattice [14]. In both cases similar mean field qualitative features are displayed; the low temperature evolution is extremely flat, but there appears a rapid change in the critical region, so that $\langle \bar{q}q \rangle^* \simeq \langle \bar{q}q \rangle$ when $\langle \Omega \rangle \simeq 0$ and $\langle \bar{q}q \rangle \simeq 0$ when $\langle \Omega \rangle \simeq 1$. A more general discussion and

given by

$$\frac{\langle \bar{q}q \rangle_*}{\langle \bar{q}q \rangle} = \tanh(M/2T) = 1 - 2e^{-M/T} + 2e^{-2M/T} + \dots \tag{12}$$

where $M = M_S/2$, despite the absence of poles in the quark propagator.

diagramatic interpretation of these issues as well as the influence of higher quark loop effects and dynamical Polyakov loop contributions will be presented elsewhere [15] providing a justification of the one quark loop approximation at least at low temperatures. There one obtains that the Polyakov loop effect can be factored out as follows[4]

$$\langle \bar{q}q \rangle^* = \sum_n \frac{1}{N_c} \mathrm{Tr}_c((-\Omega)^n) \langle \bar{q}(n\beta) q(0) \rangle \, . \tag{15}$$

This result is consistent with applying the center symmetry selection rule, Eq. (8), to the $Z(N_c)$ breaking condensate, Eq. (10), of the chiral quark model without Polyakov loops. If one now takes a suitable average on Polyakov loop configurations consistent with center symmetry, i.e., including for each such configuration all its Gribov replicas, Eq. (7) applies. Schematically, this yields

$$\langle \bar{q}q \rangle^* \sim \sum_n \langle \bar{q}(n N_c \beta) q(0) \rangle \sim \sum_n e^{-n N_c M/T} \tag{16}$$

in the confining phase. (In the above sums each term carries a weight coming from the Polyakov loop average and phase space factors.) On the other hand in the unconfining phase, where the center symmetry is spontaneously broken, the Polyakov loop is nearly unity and one recovers the standard chiral quark models results, without Polyakov loop coupling.

5 Chiral Lagrangians at finite temperature

It is interesting to construct the coupling of Polyakov loops with composite pion fields at finite temperature. Using the heat kernel techniques presented in Ref. [6] and already applied to massless QCD [7], we can obtain the lowest order chiral Lagrangian

$$\mathcal{L}_q^{(2)} = \frac{f_\pi^{*2}}{4} \mathrm{tr}_f \left(\mathbf{D}_\mu U^\dagger \mathbf{D}_\mu U + (\overline{\chi}^\dagger U + \overline{\chi} U^\dagger) \right) \tag{17}$$

where U is the non-linear transforming pseudoscalar Goldstone field, $\bar{\chi}$ the quark mass matrix and tr_f is the trace in flavor space. The pion weak decay constant, f_π^*, at finite temperature in the presence of the Polyakov loop and in the chiral limit is given by

$$f_\pi^{*2} = 4M^2 \, T \, \mathrm{Tr}_c \sum_{\hat{\omega}_n} \int \frac{d^3k}{(2\pi)^3} \frac{1}{[\hat{\omega}_n^2 + k^2 + M^2]^2} \, .$$

The full calculation of the low energy constants at order $\mathcal{O}(p^4)$ as a function of temperature and the Polyakov loop is carried out in Ref. [15]. The main feature is, similarly to $\langle \bar{q}q \rangle^*$ and f_π^*, a strong suppression $\mathcal{O}(e^{-N_c M \beta})$ at low temperatures, but an enhancement of quark thermal effects close to the chiral-deconfinement phase transition.

[4]Note that in this formula $\langle \bar{q}(n\beta) q(0) \rangle$ refers to quarks uncoupled to the Polyakov loop while in Eq. (8) it refers to quenched QCD.

6 Conclusions

We see that the coupling of the Polyakov loop to chiral quark models at finite temperature accounts for large gauge invariance and modifies in a non-trivial way the results for physical observables. On the one hand, such a coupling allows to satisfy the requirements of chiral perturbation theory at low temperatures, generating a very strong suppression at low temperatures of quark loop effects. Nonetheless, the onset of deconfinement through a non vanishing value of the Polyakov loop accounts for a chiral phase transition at somewhat similar temperatures as in the original studies where the Polyakov loop was set to one. We expect this feature to hold also in the calculation of other observables. Although these arguments do not justify by themselves the application of these chiral quark-Polyakov models to finite temperature calculations, they do show that they do not contradict basic expectations of QCD at finite temperature.

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